Let us bury the prehistoric h: arguments against using h^{-1}Mpc units in observational cosmology[†]

Ariel G. Sánchez[1, *]

[1] *Max-Planck-Institut für extraterrestrische Physik, Postfach 1312, Giessenbachstr., 85741 Garching, Germany*

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It is common to express cosmological measurements in units of h^{-1}Mpc. Here, we review some of the complications that originate from this practice. A crucial problem caused by these units is related to the normalization of the matter power spectrum, which is commonly characterized in terms of the linear-theory rms mass fluctuation in spheres of radius $8\,h^{-1}$Mpc, σ_8. This parameter does not correctly capture the impact of h on the amplitude of density fluctuations. We show that the use of σ_8 has caused critical misconceptions for both the so-called σ_8 tension regarding the consistency between low-redshift probes and cosmic microwave background data, and the way in which growth-rate estimates inferred from redshift-space distortions are commonly expressed. We propose to abandon the use of h^{-1}Mpc units in cosmology and to characterize the amplitude of the matter power spectrum in terms of σ_{12}, defined as the mass fluctuation in spheres of radius 12 Mpc, whose value is similar to the standard σ_8 for $h \sim 0.67$.

Introduction.— Common statistics used to analyse the large-scale structure of the Universe are based on observable quantities such as galaxy angular positions, redshifts, and shapes. Relating these quantities to density fluctuations on a given physical scale requires the assumption of a fiducial cosmology. To avoid adopting a specific value of the Hubble parameter, it is common practice to express all scales in units of h^{-1}Mpc, where h determines the present-day value of the Hubble parameter as $H_0 = 100\,h\,\mathrm{km\,s^{-1}Mpc^{-1}}$. At low redshift, the comoving distance $\chi(z)$, given by

$$\chi(z) = \int_0^z \frac{c\,\mathrm{d}z'}{H(z')}, \tag{1}$$

can be approximated as

$$\chi(z) \approx \frac{c}{H_0} z. \tag{2}$$

Then, using h^{-1}Mpc units effectively yields a distance independent of any assumption of the fiducial cosmology. This approach was applied to the analysis of the first galaxy redshift surveys [1, 2], which probed only small volumes. However, this practice has continued until the analysis of present-day galaxy surveys such as the Baryon Oscillation Spectroscopic Survey (BOSS) [3], which cover much larger volumes and where the use of the distance – redshift relation of Eq. (1) requires the assumption of a full set of fiducial cosmological parameters.

*arielsan@mpe.mpg.de

[†]The title of this letter is a reference to the provocative speech given by Nobel laureate Gabriel García Marquez during the First International Congress of the Spanish Language, where he made a plea to simplify Spanish orthography by, among other things, getting rid of the soundless letter h using the sentence "enterremos las haches rupestres".

As cosmological observations are expressed in h^{-1}Mpc units, theoretical predictions also follow the same approach. These units obscure the true dependence of the matter power spectrum, $P(k)$, on h. Moreover, the amplitude of $P(k)$ is often characterized in terms of the rms linear perturbation theory variance in spheres of radius $r = 8\,h^{-1}$Mpc, commonly denoted as σ_8. For models with different values of h, this corresponds to different reference scales in Mpc. Furthermore, σ_8 is often constrained by cosmological data that provide different posterior distributions on h, which means that the inferred values probe the amplitude of density fluctuations on different scales. In the following sections we will discuss the implications and misconceptions related with the use of h^{-1}Mpc units, and how they can be avoided.

Impact of the fiducial cosmology.— Three-dimensional galaxy clustering measurements depend on the particular cosmology used to transform the observed redshifts into comoving distances. Any difference between this fiducial cosmology and the true underlying one gives rise to the so-called Alcock-Paczynski (AP) distortions [4]. This geometric effect distorts the inferred components parallel and perpendicular to the line-of-sight, $s_\parallel$ and $s_\perp$, of the total separation vector $\mathbf{s}$ between any two galaxies as [5, 6]

$$s_\parallel = q_\parallel s'_\parallel, \tag{3}$$

$$s_\perp = q_\perp s'_\perp, \tag{4}$$

where the primes denote the quantities in the fiducial cosmology and the scaling factors are given by

$$q_\parallel = \frac{H'(z_\mathrm{m})}{H(z_\mathrm{m})}, \tag{5}$$

$$q_\perp = \frac{D_\mathrm{M}(z_\mathrm{m})}{D'_\mathrm{M}(z_\mathrm{m})}, \tag{6}$$

where $H(z)$ is the Hubble parameter, $D_\mathrm{M}(z)$ is the comoving angular diameter distance, and z_m is the effective

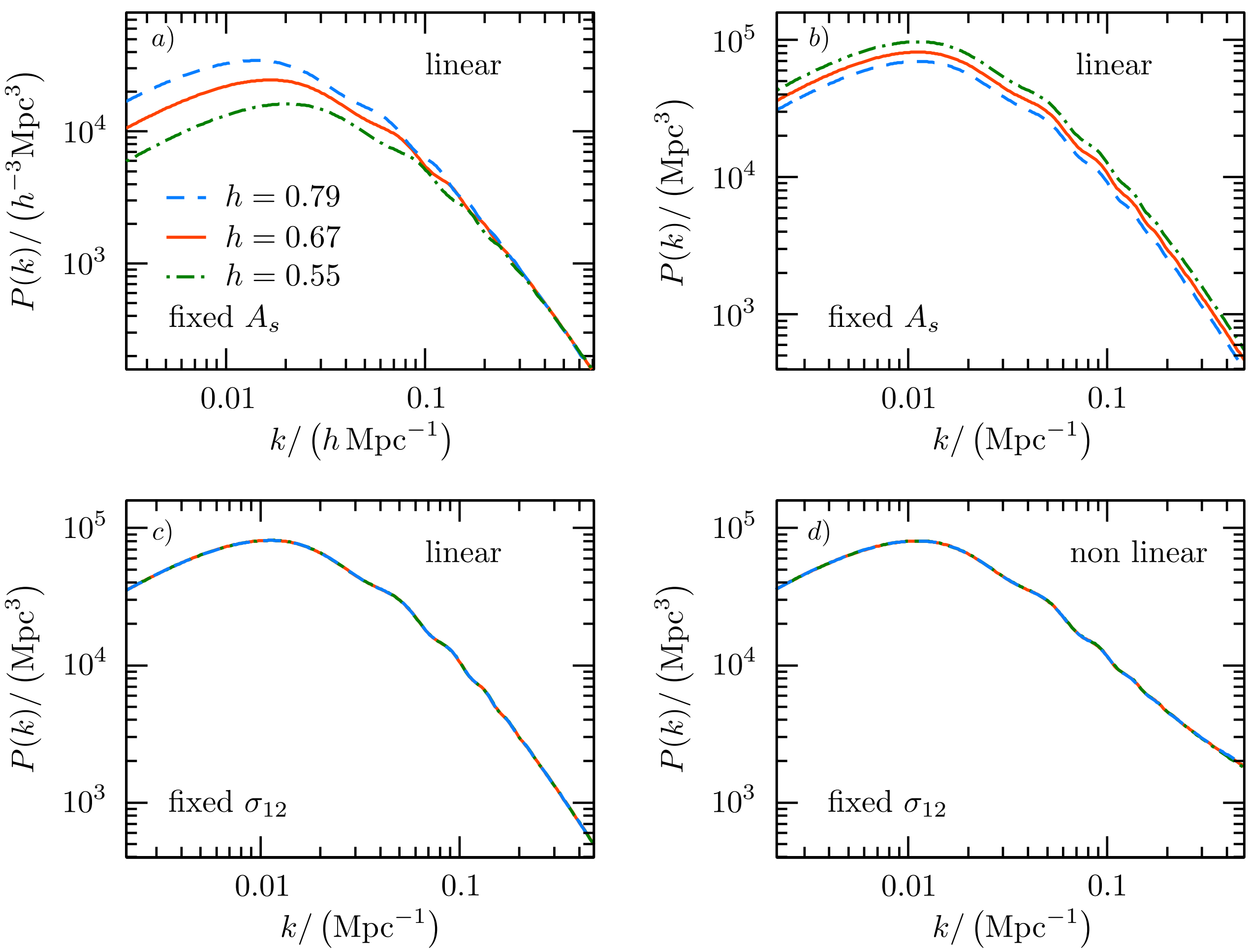

FIG. 1: Panel *a*): linear matter power spectra at $z = 0$ of three ΛCDM models defined by identical cosmological parameters with the exception of h, expressed in h^{-1}Mpc units. Panel *b*): the same power spectra of panel *a* shown in units of Mpc. Panel *c*): the power spectra of the same models of panel *b* but with their values of $A_{\rm s}$ adapted to produce the same value of σ_{12}. Panel *d*): non-linear matter power spectra corresponding to the same models of panel *c*.

redshift of the galaxy sample. If the clustering measurements are expressed in h^{-1}Mpc , the quantities appearing in Eqs. (5) and (6) must also be computed in these units.

AP distortions are exploited as a source of cosmological information, most notably by means of measurements of the baryon acoustic oscillation (BAO) feature [7–12]. These measurements are based on the use of the sound horizon scale at the drag redshift, $r_{\rm d}$, as a standard ruler. The information recovered from these measurements is often expressed in terms of the BAO shift parameters

$$\alpha_\parallel = \frac{H'(z_{\rm m})}{H(z_{\rm m})}\,\frac{r'_{\rm d}}{r_{\rm d}}, \tag{7}$$

$$\alpha_\perp = \frac{D_{\rm M}(z_{\rm m})}{D'_{\rm M}(z_{\rm m})}\,\frac{r'_{\rm d}}{r_{\rm d}}. \tag{8}$$

If the values of $H(z)$ and $D_{\rm M}(z)$ are expressed in h^{-1}Mpc units, the sound horizon in the true and fiducial cosmology must be given in these units as well.

As the fiducial sound horizon $r'_{\rm d}$ can be predicted in Mpc as a function of the physical baryon and matter densities, $\omega_{\rm b}$ and $\omega_{\rm c}$, its mapping into h^{-1}Mpc requires the assumption of a value of h. Avoiding this assumption was the original motivation of using h^{-1}Mpc units. As the procedure required to account for the impact of the fiducial cosmology on clustering measurements is essentially the same whether they are expressed in h^{-1}Mpc units or if a value of h is explicitly assumed, there is no advantage in using the traditional units with regard to BAO analyses.

The normalization of the power spectrum.— We now focus on the complications associated with the use of h^{-1}Mpc units when computing model predictions for clustering measurements. Panel a of Fig. 1 shows the linear matter power spectra at $z = 0$ of three different ΛCDM models expressed in h^{-1}Mpc units, computed using CAMB [13]. These power spectra have been obtained by fixing all physical density parameters, ω_i, as well as the amplitude and spectral index of the scalar mode, $A_{\rm s}$ and $n_{\rm s}$, and varying only the value of h. Panel b of Fig. 1 shows the same power spectra in units of Mpc, without introducing the traditional h factors. The power spectra

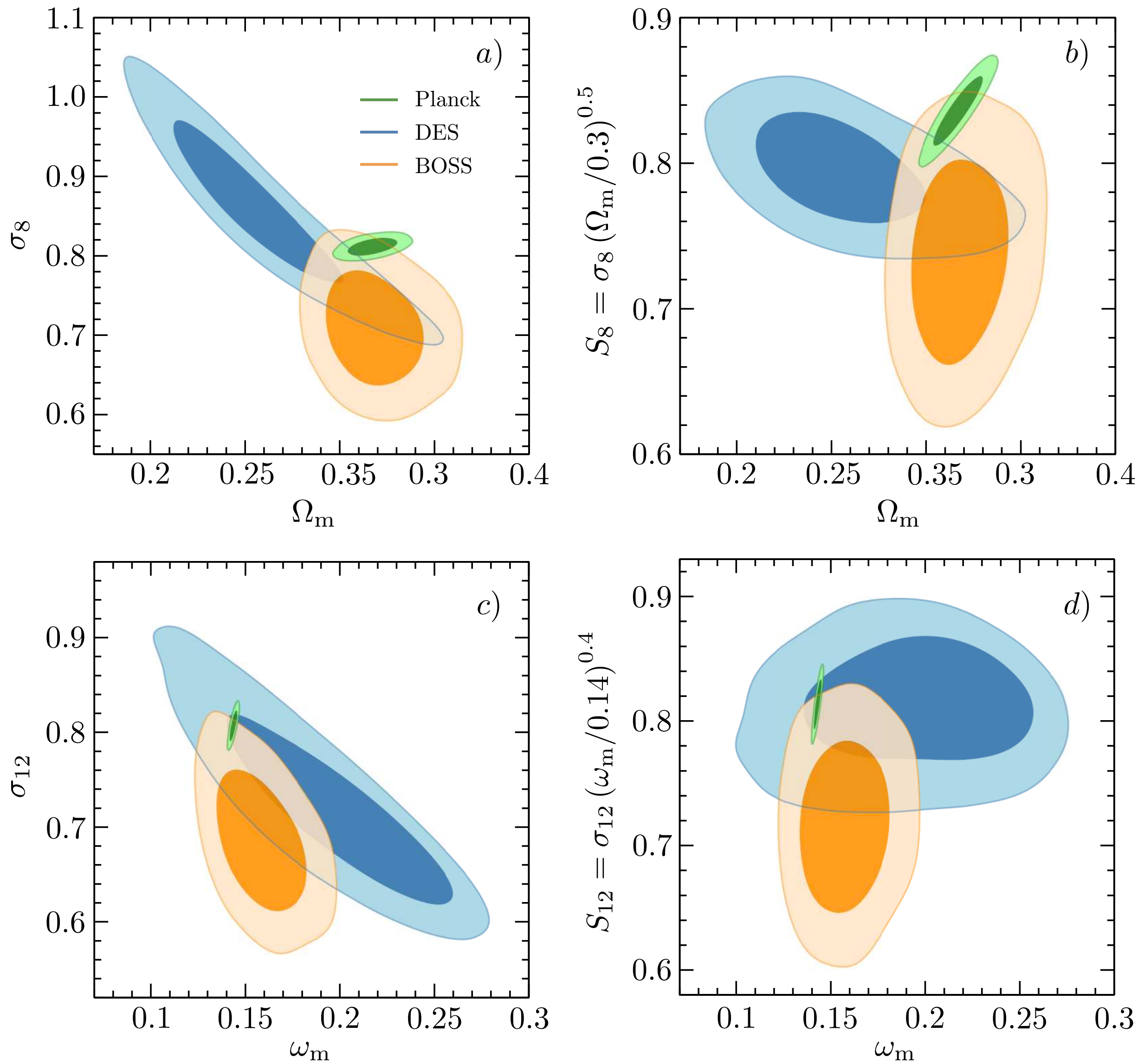

FIG. 2: Two-dimensional 68% and 95% constraints recovered from Planck (green), the 3×2pt analysis of DES (blue), and BOSS (orange) under the assumption of a ΛCDM cosmology on the parameters $\Omega_m - \sigma_8$ (panel *a*), $\Omega_m - S_8 = \sigma_8\,(\Omega_m/0.3)^{0.5}$ (panel *b*), $\omega_m - \sigma_{12}$ (panel *c*), and $\omega_m - S_{12} = \sigma_{12}\,(\omega_m/0.14)^{0.4}$ (panel *d*).

of these models have the same shape, with the BAO feature appearing at the same scales, and differ only in their amplitude. Expressing these power spectra in h^{-1}Mpc units introduces a spurious dependency on the value of h, which actually only affects the overall clustering amplitude when they are expressed in Mpc.

For a ΛCDM universe, the amplitude of the power spectrum is controlled by both h and A_s. The joint effect of the two parameters is usually described in terms the linear perturbation theory variance in spheres of radius $r = 8\,h^{-1}\,$Mpc, σ_8. A problem with this quantity is that $8\,h^{-1}\,$Mpc corresponds to a different scale for models with different values of h. Normalizing the power spectra of Fig. 1 to the same value of σ_8 would actually aggravate the amplitude mismatch.

A better choice to describe the degenerate effect of h and A_s is to normalize the power spectrum in terms of the mass variance at a given scale in Mpc. For simplicity, we propose to use σ_{12}, defined as the rms linear theory variance at $r = 12\,$Mpc. For models with $h \simeq 0.67$ as suggested by current CMB observations, $8\,h^{-1}\,$Mpc $\simeq$ 12 Mpc and σ_{12} has a similar value to the standard σ_8. However, these parameters differ for other values of h. Panel *c* of Fig. 1 shows the power spectra of the same models of panel *b* with their values of A_s modified to produce the same value of σ_{12}, which are now identical. This shows that the perfect degeneracy between h and A_s is more naturally described in terms of σ_{12} than the standard σ_8.

Panel *d* of Fig. 1 shows the predictions for the non-linear $P(k)$ of the same models shown in panel c, computed using the halofit formalism [14]. The observed agreement, with only differences of a few percent at high k, shows that σ_{12} is a more adequate parameter to characterize the non-linear evolution of $P(k)$ than σ_8.

Revising the σ_8 tension.— It is common to include σ_8 in the lists of parameters that are constrained by cosmological observations. These constraints have been the focus of significant attention in recent years as the value of σ_8 preferred by Planck CMB data [15] under the assumption of a ΛCDM universe is higher than the estimates derived from all recent weak lensing (WL) datasets [16–19] and the abundance of galaxy clusters [20]. Recent analyses of anisotropic clustering measurements from BOSS have found similar differences with the CMB predictions [21–23]. These discrepancies, which have been dubbed the "σ_8 tension", are illustrated in panel a of Fig. 2, which shows the constraints on $\Omega_{\rm m}$ and σ_8 recovered from the final Planck CMB data [15], the auto- and cross-correlations between the cosmic shear and galaxy positions (the so-called 3×2pt analysis) from the Dark Energy Survey (DES) [24], and the galaxy clustering measurements from BOSS [21, 25]. These results assume a ΛCDM cosmology with the same wide uniform priors as in the analysis of [21]. Panel b of Fig. 2 shows these constraints expressed in terms of $S_8 = \sigma_8\,(\Omega_{\rm m}/0.3)^{0.5}$, which is often used to describe WL constraints. For the same values of $\Omega_{\rm m}$ preferred by Planck, the low-redshift data prefer lower values of S_8 than the CMB constraints.

As discussed before, a drawback of using σ_8 to characterize the amplitude of the power spectrum is its dependence on h. As each dataset shown in Fig. 2 provides different constraints on h, the scales corresponding to $8\,h^{-1}$Mpc are also different. While Planck gives $8\,h^{-1}{\rm Mpc} = (11.8\pm 0.21)$ Mpc, the low-redshift data of BOSS and DES imply $8\,h^{-1}{\rm Mpc} = 11.38^{+0.77}_{-0.71}$ Mpc and $8\,h^{-1}{\rm Mpc} = 9.3^{+2.2}_{-1.3}$ Mpc, respectively. Although these results are consistent within 1σ, their difference implies that the values of σ_8 recovered from these data cannot be directly compared as they characterize the amplitude of density fluctuations on different scales.

These issues can be avoided by using as a reference the value of σ_{12}, which is independent of the constraints on h obtained from a particular dataset. Panel c of Fig. 2 shows the constraints in the $\omega_{\rm m} - \sigma_{12}$ plane recovered from the same datasets. We use the physical density $\omega_{\rm m}$ instead of the density parameter $\Omega_{\rm m}$ as the former is the most relevant quantity to characterize the shape of $P(k)$. When expressed in terms of σ_{12}, the constraints inferred from DES and Planck are in agreement. BOSS data prefer lower values of σ_{12}. These constraints are largely driven by a degeneracy between this parameter and the scalar spectral index $n_{\rm s}$, which is not well constrained by BOSS data alone. Panel d of Fig. 2 shows these results in terms of the parameter combination $S_{12} = \sigma_{12}\,(\omega_{\rm m}/0.14)^{0.4}$, which matches the degeneracy between $\omega_{\rm m}$ and σ_{12} recovered from DES data. When the amplitude of $P(k)$ is measured at a scale in Mpc, low and high-redshift data show consistent constraints. Planck and DES imply $S_{12} = 0.815 \pm 0.026$ and $S_{12} = 0.812^{+0.068}_{-0.077}$ respectively, while BOSS gives $S_{12} = 0.718^{+0.094}_{-0.090}$, which are all consistent within 1σ.

A detailed assessment of the consistency between Planck and low-redshift data, including other WL surveys or the abundance of galaxy clusters, is out of the scope of this *letter*. However, such studies should be based on σ_{12} as an indicator of the amplitude of density fluctuations.

The growth rate of cosmic structures.— The analysis of redshift-space distortions (RSD) based on anisotropic two-point clustering measurements is considered as one of the most robust tools to constrain the growth rate of structures across cosmic time [26]. In linear perturbation theory, the two-dimensional galaxy power spectrum, $P_{\rm g}(k,\mu,z)$, is related to the real-space matter power spectrum by [27]

$$P_{\rm g}(k,\mu,z) = \left(b(z) + f(z)\mu^2\right)^2 P(k,z). \qquad (9)$$

where μ represents the cosine of the angle between $\mathbf{k}$ and the line-of-sight direction, $b(z)$ is the galaxy bias factor and $f(z)$ is the linear growth rate parameter. Eq (9) can be written as

$$P_{\rm g}(k,\mu,z) = \left(b\sigma_8(z) + f\sigma_8(z)\mu^2\right)^2 \frac{P(k,z)}{\sigma_8^2(z)}. \qquad (10)$$

If $\sigma_8^2(z)$ described the amplitude of the power spectrum, the ratio $P(k,z)/\sigma_8^2(z)$ would only depend on the parameters that control its shape. In this case, the anisotropies in $P_{\rm g}(k,\mu,z)$ would only depend on the combination $f\sigma_8(z)$, which is the quantity in which the results of RSD analyses are expressed. However, the ratio $P(k,z)/\sigma_8^2(z)$ depends on the actual value of h. We can then expect to obtain different results depending on the assumed value of h or when this parameter is marginalized over.

In most RSD studies, anisotropic clustering measurements are used to constrain the values of $f\sigma_8(z)$ and the BAO shift parameters of Eqs. (7) and (8), together with additional nuisance parameters, while the cosmological parameters that determine the shape and amplitude of the matter $P(k)$, including h, are kept fixed. To test this approach we used linear perturbation theory to obtain predictions of the Legendre multipoles $P_{\ell=0,2,4}(k)$ of a synthetic galaxy sample roughly matching the volume, bias, and number density of the final BOSS CMASS sample [28] and used a Gaussian prediction for their corresponding covariance matrix [29]. We then used these data to constrain the combinations $b\sigma_8(z)$ and $f\sigma_8(z)$, as well as the BAO shift parameters.

Panel a of Fig. 3 shows the constraints in the $b\sigma_8(z)$ – $f\sigma_8(z)$ plane obtained when both $A_{\rm s}$ and h are kept fixed (orange), which corresponds to the standard RSD analysis, when $A_{\rm s}$ is varied while h is kept fixed (green), and the more general case in which both $A_{\rm s}$ and h are

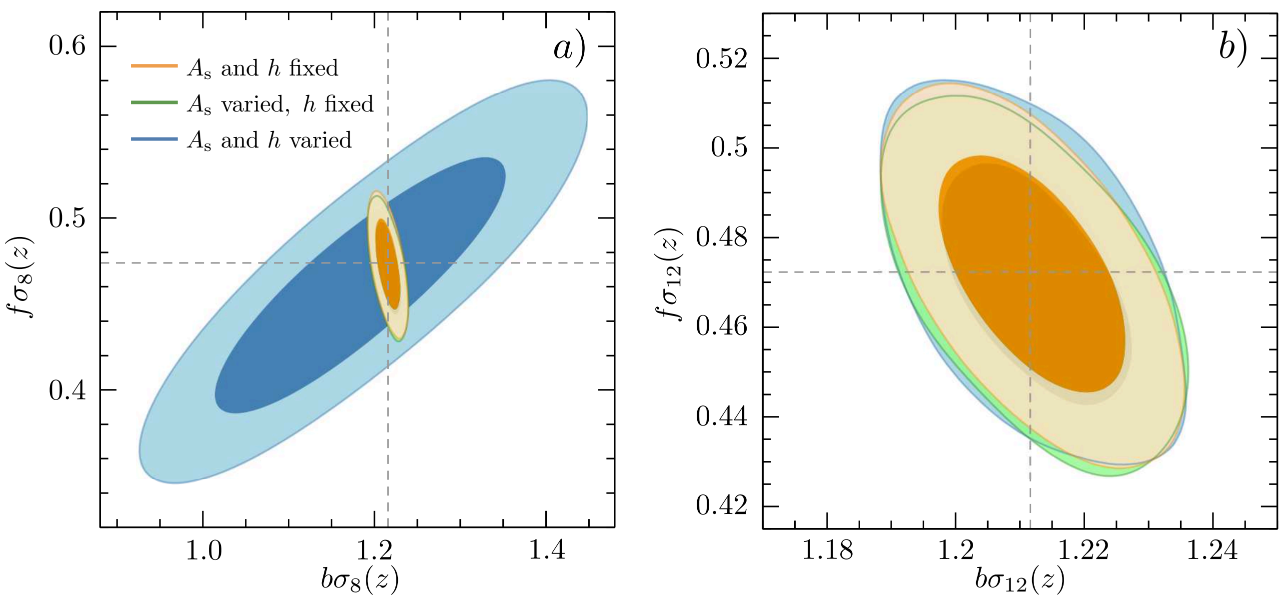

FIG. 3: Panel a): constraints on the parameters $b\sigma_8(z)$ and $f\sigma_8(z)$ derived from synthetic Legendre multipoles $P_{\ell=0,2,4}(k)$ corresponding to a galaxy sample matching the volume, bias, and number density of the BOSS CMASS sample. The contours correspond to the standard analysis in which A_s and h are kept fixed (orange), when A_s is varied and h is fixed (green), and when both parameters are varied (blue). Panel b): same constraints as panel a but expressed in terms of $b\sigma_{12}(z)$ and $f\sigma_{12}(z)$.

allowed to vary (blue). The dashed lines correspond to the true values of these parameters. When only A_s is varied, the constraints follow the expected degeneracies defined by constant values of $b\sigma_8(z)$ and $f\sigma_8(z)$, leading to identical results to the ones obtained when it is fixed. However, when h is also varied, their degenerate effect on the amplitude of $P(k)$ is not fully captured by $\sigma_8(z)$ and the obtained constraints deviate significantly from the results recovered in the standard case. This shows that RSD data can only constrain $f\sigma_8(z)$ when a fixed value of h is assumed. Without this strong assumption, the information recovered from RSD fits is not well described as a measurement of $f\sigma_8(z)$.

Panel b of Fig. 3 shows the same constraints as in panel a, but expressed in terms of $b\sigma_{12}(z)$ and $f\sigma_{12}(z)$. In terms of these variables, the results are the same irrespective of whether A_s or h are kept fixed or marginalized over. This shows that the combination $f\sigma_{12}(z)$ provides a more correct description of the information retrieved from the standard RSD analyses. Note that, although for $h \simeq 0.67$ the numerical values of $f\sigma_{12}(z)$ and $f\sigma_8(z)$ are similar, their cosmological implications are different.

Conclusions.— In this *letter* we have reviewed a number of drawbacks associated with the use of h^{-1}Mpc units in cosmology. An important problem due these units is related to the normalization of the matter power spectrum in terms of the standard σ_8. This parameter does not correctly capture the impact of h on the amplitude of $P(k)$, which is better described in terms of a reference scale in Mpc. A convenient choice is 12 Mpc, which results in a mass variance σ_{12} with a similar value to the standard σ_8 for $h \sim 0.67$.

We have shown that the agreement between low- and high-redshift data should be quantified in terms of σ_{12}, which is independent of the constraints on h provided by any given dataset. The results of standard RSD analyses are more correctly expressed in terms of $f\sigma_{12}(z)$, which changes the cosmological implications of most growth-rate measurements obtained so far. In the coming years, new large-volume surveys [30, 31] will challenge our ability to obtain robust cosmological constraints. The arguments presented here indicate that we should abandon the use of the traditional h^{-1}Mpc units in the analysis of these new high-quality datasets, and to replace σ_8 by the analogous σ_{12} as a better quantity to characterize the amplitude of density fluctuations.

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