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IMPROVING MEASUREMENTS OF DARK ENERGY WITH TYPE IA SUPERNOVAE BY USING THE LARGEST PHOTOMETRIC SAMPLES

Brodie Popovic

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1. INTRODUCTION

In 1923, Edwin Hubble discovered four Cepheid variable stars in the Andromeda Nebula. Using the period-luminosity relationship for Cepheids discovered by Leavitt & Pickering (1912) the decade before, he proved that the universe was far larger than our local galaxy. Objects that emit radiation typically do so in obeisance to the inverse square law - that is to say, the brightness we observe is related to the absolute luminosity by $b = L/(4\pi d^2)$, where b is the apparent brightness, L is the luminosity, and d is the distance to the object. Given the astronomically large distances that cosmologists deal in, this relationship is often expressed logarithmically in magnitudes (a measure of luminosity):

$$\mu = m - M = 5\ \log_{10}(\frac{d}{10}) \tag{1}$$

with μ being the distance modulus, the difference between apparent magnitude m and absolute magnitude M. Knowing the absolute luminosity of Cepheids, Hubble was able to compare their observed magnitudes and derive distances. Only a few years later, Hubble made an even more important discovery. The propensity of galaxies to redshift was known, discovered by Vesto M. Slipher (Slipher 1917), but not well understood. Redshift arises from the Doppler Effect, such that objects moving away from an observer appeared redder than they would at rest-frame. Astronomers and cosmologists would take to labelling redshift as z - which can be measured with the wavelength of light as $z = (\lambda_{\rm obs} - \lambda_{\rm emit})/\lambda_{\rm emit}$. While nominally unitless, this shift in wavelength can be transformed into a velocity, and is such considered synonymous with velocity. Assuming some universal expansion or contraction can further turn z into a stand-in for time - the larger the redshift, the further back in the history of the universe the event occurred.

Hubble showed that the redshift of the galaxy was proportional to its distance from Earth. With only a handful of nebulae, shown in figure 1, he had proved something quite curious - the universe seemed to be expanding.

Hubble found in his observations that the relationship between the redshift and distance for these objects seemed to obey a simple relationship (Hubble 1929) relating velocity v and distance d with a constant H_0:

$$v = H_0 \times d \tag{2}$$

Around the same time, Albert Einstein published his General Theory of Relativity and posited an odd constant in order to keep the size of the universe static - a cosmological constant, Λ. Upon hearing Hubble's discovery that the universe was expanding, Einstein struck Λ from the record and called it his "greatest mistake". Other scientists had also noticed this, and Alexander Friedmann, of Friedmann-Lemaître-Robertson-Walker (FLRW) fame, suggested that a static universe was unstable and that an expanding model was more likely. One of the FLRW solutions to the Einstein Field Equations concerned expansion:

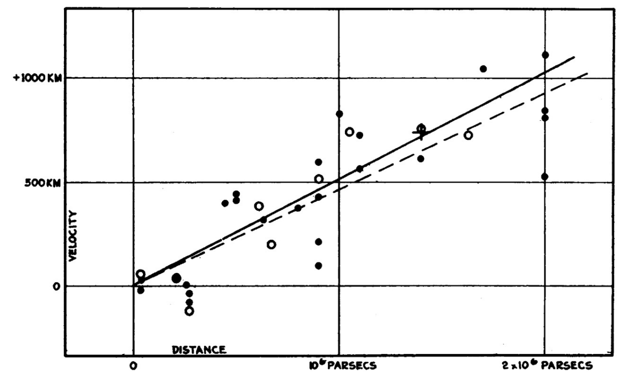

Fig. 1.— The velocities of several Cepheids and nebulae are plotted as a function of distance from earth. Black disks are individual nebulae and white circles are group-averaged measurements. Later iterations will begin labelling velocity as redshift, and the 'redshift vs distance' plot is named a Hubble Diagram. This Hubble diagram shows a clear trend of expansion.

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + \frac{3p}{c^2}) + \frac{\Lambda c^2}{3} \tag{3}$$

where 'scale factor' $a = (1+z)^{-1}$, G is Newton's gravitational constant, c is the speed of light, ρ is density, p is pressure, and Λ is Einstein's cosmological constant. Assuming that the universe is undergoing adiabatic expansion gives rise to ρ and p.

By the 1960s, with the discovery of the Cosmic Microwave Background Radiation, the Big Bang model was widely favoured over a steady-state universe model. Cosmologists at the time dropped the cosmological constant term from equation 3. There was only one question left - how rapidly is this expansion slowing down?

Theorists developed a deceleration parameter, $q \equiv -(\ddot{a}a)/\dot{a}^2$, that characterised the rate of deceleration. This can be greatly simplified by separating equation 3 into component parts (matter, radiation, curvature, and the cosmological constant) and assuming that each component acts as a perfect fluid that obeys the relationship $w \equiv p/\rho$.

Using the conservation of energy, a flat universe (the case for which has been strongly made by modern measurements and inflation theory), and a constant w for each component, it is the case that $\rho \propto a^{3(1+w_i)}$. Cosmologists find it simplest to use the critical density $\rho_c = 3H^2/8$, the average density of matter required for a universe that ceases to expand after an infinite amount of time. Using ρ_c allows us to define equation 3 in terms of

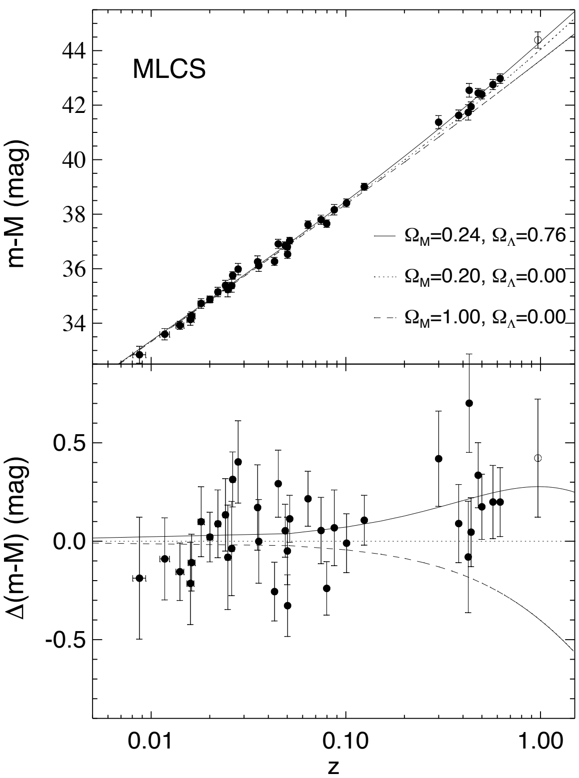

FIG. 2.— A more modern Hubble Diagram, comparing redshift and distance modulus $m - M = \mu$. Plotted alongside the data are three proposed universe models. The bottom plot shows the difference between the data and the three proposed models. The SNIa shown here are fainter (and therefore further) than any model that does not have a significant dark energy component (Ω_Λ).

relative densities, and indeed, setting $\Omega_i \equiv \rho_i/\rho_c$, equation 3 becomes

$$q = \Omega_{\rm rad}(z) + \frac{1}{2}\Omega_{\rm m}(z) + \frac{1+3w}{2}\Omega_\Lambda(z) \qquad (4)$$

The greatest weakness of Cepheid stars as a standard candle is their low luminosity. While effective probes of nearby cosmology, they are precipitously uncommon at $z = 0.01$, limiting their efficacy as a cosmological probe. But as early as the 1970s, interest in supernovae as standard candles was growing. Significant progress was made in the 1980s on determining the nature of these explosions. One particular type, designated type Ia supernovae (SNIa), was modeled as the conflagration of a white dwarf upon reaching the Chandrasekhar mass. Their extreme (~ -19.5 mag) and apparently uniform luminosity made them an attractive tool for finding evidence of cosmic deceleration.

The early 90s were dominated with paring down the final vestiges of uncertainty surrounding SNIa. Assumptions that all SNIa had the same intrinsic colour were discarded due to inconsistencies with dust reddening (Branch & Tammann 1992), deep-set intrinsic scatter was discovered and partially mitigated with initial attempts at standardisation (Phillips 1993). This work came to a fore in the latter half of the 1990s with the development of the first SNIa standardisation method, the Multi-Color Light Curve Shape (MLCS) method, and two key papers, Riess et al. (1998) and Perlmutter et al. (1999). With 10s of spectroscopically confirmed SNIa, figure 2 shows not only that the universe was expanding, it is accelerating in that expansion.

This strong claim is verified with equation 4. Riess et al. (1998) and Perlmutter et al. (1999) probe the deceleration parameter today, q_0. The local radiation contribution, $\Omega_{\rm rad}(0)$, is negligible (as we are alive and not irradiated). Assuming that $w = -1$, $q_0 = 1/2\ \Omega_{\rm matter} - \Omega_\Lambda$ Figure 2 shows the best fit to the data gives $q_0 \approx -1$. Historically, a deceleration parameter of positive value indicates deceleration - the measurement of a negative value is indicative of accelerating expansion.

2. SNIA COSMOLOGY TODAY

Since 1999, cosmology done with SNIa has improved by leaps and bounds. Spectroscopy is measurement de rigueur for its precise measurements and resistance to the inimical core collapse contamination. Since the development of MLCS, it has been eclipsed by the SALT2 model developed by Guy et al. (2010). Both measure the light-curves of SNIa and work to standardise the peak brightness. This peak brightness is related to the decline rate of the light curve and relative fluxes between different filters present in telescopes. The SALT2 model became the de facto standard for SNIa standardisation, modifying the Tripp distance estimator (Tripp 1998)

$$\mu = mB + \alpha x_1 - \beta c - M_0 \qquad (5)$$

to include fitted parameters related to light-curve shape (related to the decline rate; x_1), spacing (flux difference between filters; c) and peak brightness (mB). It also includes global nuisance parameters α and β, and the absolute SNIa luminosity offset term M_0 (-19.5mag) . Plotting this distance modulus μ against the measured redshift of the supernova shows the evolution of distance with redshift, which can be taken further to measure the dark energy equation-of-state, w, and $\Omega_{\rm matter}$, the fractional energy contribution of matter to the observed universe.

The late 2000s and early 2010s were dominated by surveys such as the Sloan Digital Sky Survey (SDSS), Supernova Legacy Survey (SNLS), a low redshift targeted survey LOWZ, Pan-STARRs, and most recently, the Dark Energy Survey (DES).

Each has collected 100s of spectroscopic SNIa and subsequently driven the statistical uncertainty lower and lower. The current most accurate measurement, done by Scolnic et al. (2018) and shown in figure 3, has $w = 1.026 \pm 0.041$, collected from a wide array of surveys across multiple decades. The statistical uncertainty accounts for most of this error, at 0.031.

SNIa cosmology will soon arrive at the bounds of statistical error. But it seems unlikely that spectroscopy will bring us there. Newer telescopes such as LSST and WFIRST will not have the spectroscopic capacity of previous surveys. The relative sparsity of SNIa across the night sky makes multi-object spectroscopy ineffective, and the cadence of observations needed for well-measured light-curves prohibits cheap single-object spectroscopy. LSST is projected to observe on the order of 100,000s of SNIa. This is orders of magnitude more than our cur-

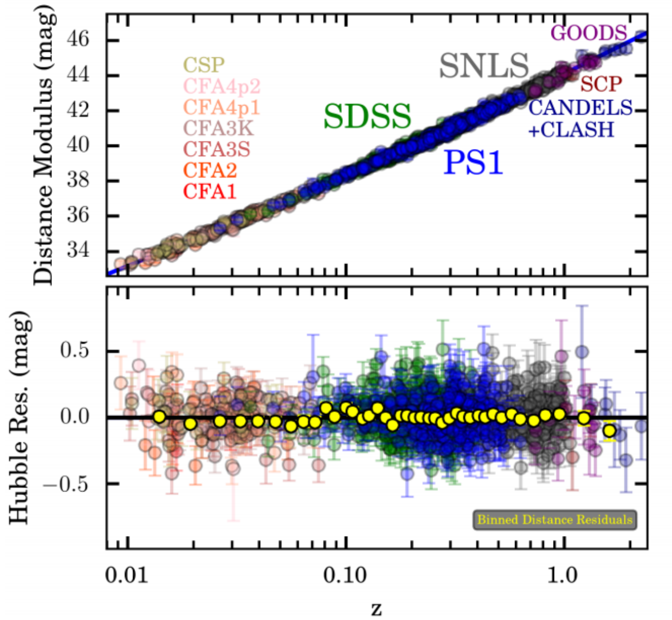

FIG. 3.— A modern Hubble Diagram from Scolnic et al. (2018). Log z is plotted against distance modulus for a collection of different SNIa surveys. Surveys are colour-coded dots, and yellow solid dots indicate average values. This dataset gives the most precise supernova measurement of w to date.

rent best collections of samples, all from a single survey. But photometric samples have their own unique uncertainties, most notably core collapse contamination and redshift measurements.

These spectroscopic surveys were contemporary to a slew of developments to further standardise SNIa. A platform for rigorous simulations of supernovae (the Supernova Analysis software, or SNANA) was developed by Kessler et al. (2009). Simulating large surveys provides an avenue for training classifiers, correcting biases, and testing new methods where truth values are known. One such method was developed by Kessler & Scolnic (2017), known as BEAMS with Bias Corrections (BBC). By simulating a large volume of SNIa, the BBC method added new terms to the Tripp equation,

$$\mu = (m_B - \bar{\delta}_{m_B}) + \alpha(x_1 - \bar{\delta}_{x_1}) - \beta(c - \bar{\delta}_c) - M_0 \quad (6)$$

where $\bar{\delta}$ are the new bias corrections. These corrections are found by averaging the corrections in a 5-Dimensional space of $\{z, x_1, c, \alpha, \beta\}$ comprised of simulated SNIa. By tracking how the observed values of the simulated SNIa diverged from their input values in each 5D cell, BBC is more accurately able to account for reddening and intrinsic noise.

After the application of the corrections, potential contamination due to core collapse supernovae can be marginalised over with the Bayesian Estimation Applied to Multiple Species (BEAMS) method by Hlozek et al. (2012). BEAMS utilises classifiers to model two independent populations. Knowing that core collapse SNe are typically less luminous and therefore have a higher magnitude, BEAMS uses classifier output as a likelihood to marginalise over probably core collapse, decreasing their effect on cosmological measurements. The use of a Taylor expanded likelihood function in the original BEAMS formalism was replaced in the BBC method with an insitu method using simulated core collapse to map their probability in distance-residual space.

These bias corrections could be incorporated with newly found parent distributions of fitted properties discovered in Scolnic & Kessler (2016). By modeling the migration of simulated c and x_1 values from generation to observation, they were able to model an asymmetric Gaussian distribution of likely values that, when observed through the course of a simulation, resulted in a match to real data distributions.

Sullivan et al. (2010) found that SNIa in host galaxies with stellar mass $> 10 M_\odot$ are intrinsically brighter than their counterparts in lower mass galaxies, observed with 4σ confidence. While this relationship, known as the mass step, or γ, was discovered first in SNLS, it has held in the SDSS, LOWZ, PS1, and DES samples. They also found other SNIa properties also correlate with host galaxy mass, notably c and x_1. These correlations are shown in figure 8. The x_1 parameter more strongly correlates with host galaxy stellar mass than c.

Years later, Smith et al. (2020) discovered that including these correlations would bias the BBC method's recovery of γ by $\sim 30\%$.

This shift to photometric samples presents an exciting opportunity in the golden age of cosmology. With new challenges come new opportunities, new problems, and new sources of funding. I have already made good progress in addressing some of these challenges, and am well positioned to continue to contribute to the field.

3. MY CONTRIBUTIONS

3.1. *Mis-associating host galaxies*

Moving away from spectroscopic identification of SNIa engenders a host of potential problems. The one that comes to mind first is determining redshift. Galactic redshift measurements are more accurate than those derived solely from the SNIa. SNIa spectral features are due to absorption profiles of different materials in the star, compared to the cleaner emission lines of galaxies. For a spectroscopic survey, having both host galaxy and SNIa redshifts allows for a solid consistency check. In photometric samples, we still need to have host galaxy redshifts, but spectroscopic measurements can happen well after the time-frame of the SNIa. Mis-associations can cause large scatter across the Hubble diagram, significantly biasing w measurements. This necessitates a more rigorous methodology for determining host galaxies. Gupta et al. (2016) present weighing galaxy shape/orientation with angular separation. The angular separation ($\Delta\theta$) is divided by the effective radius of the galaxy at the angle closest to the supernovae (DLR) to give $d_{\rm DLR}$. In Popovic et al. (2019), I present a method of determining potential host galaxies shown in figure 4. We can determine the likely host galaxy by choosing the smallest $d_{\rm DLR}$ option, and measure confusion between potential hosts with $r_{\rm DLR}$, the ratio between the smallest and second smallest $d_{\rm DLR}$ for a given SNe. Furthermore, paired with rigorous simulations, we can use this method to estimate the mis-association rate in our survey. This methodology has now been included in SNANA by default, and will be used in the DES 5 year cosmological measurement.

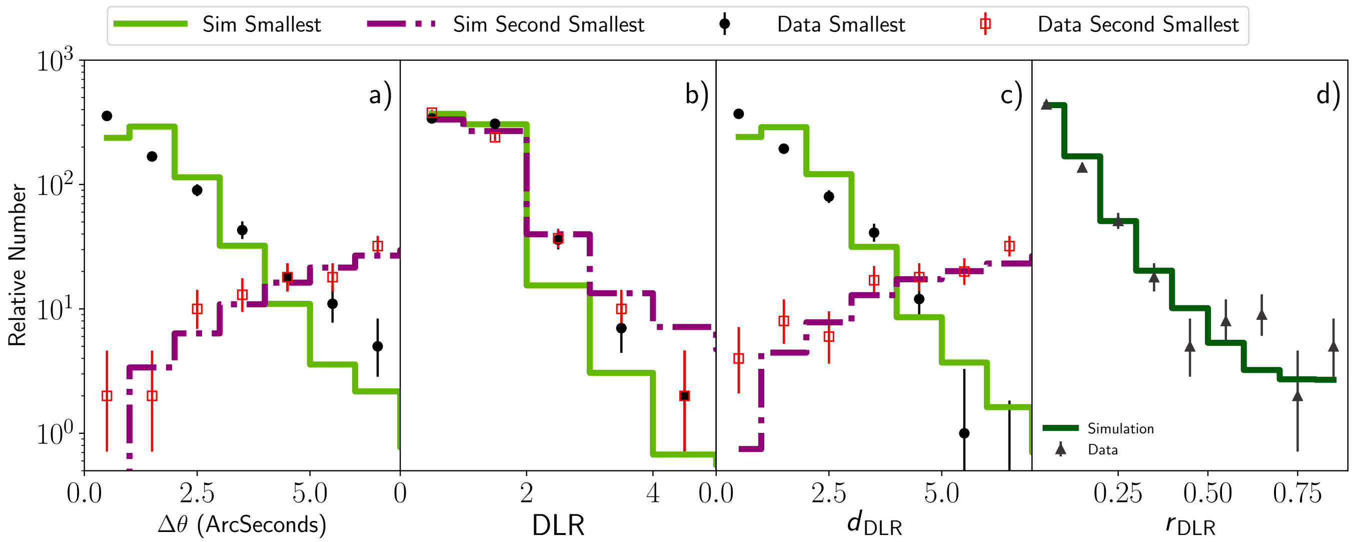

FIG. 4.— Each distribution is shown separately for the smallest and second smallest $d_{\rm DLR}$ term. $\Delta\theta$ is the angular separation, DLR is the Directional Light Radius, or effective radius of the galaxy at the angle closest to the supernova, and $d_{\rm DLR}$ is $\Delta\theta/d_{\rm DLR}$. The ratio between the smallest and second smallest $d_{\rm DLR}$ for a given supernova is $r_{\rm DLR}$. Matching these distributions ensures good agreement for $r_{\rm DLR}$. The host galaxy with the smallest $d_{\rm DLR}$ is considered the likely one, and increasing $r_{\rm DLR}$ can indicate confusion.

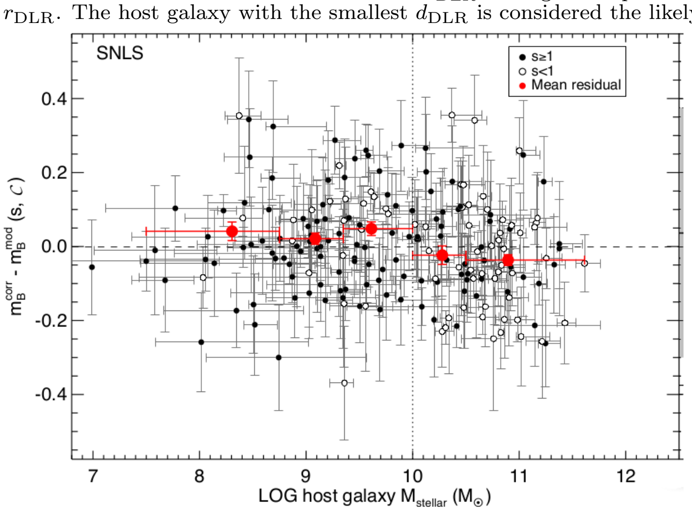

FIG. 5.— Evidence of a mass-based step function in the SNLS data set. The x-axis is the log of the host galaxy stellar mass, and the y-axis is the difference between the data and predicted cosmology. The observed difference in magnitudes across host mass is referred to as γ. The step function is observed with a 4σ confidence; a linear fit with 3.3σ confidence. Further studies lend stronger credence to a step function.

It is also possible to marginalise over mis-associated SNIa and mitigate their potential contamination in our measurements. Roberts et al. (2017) describe an application of the BEAMS formalism to changing redshift values rather than the original luminosity change. Replacing the Gaussian likelihood map suggested by Roberts et al. (2017) with the $r_{\rm DLR}$ term, it would be possible to marginalise over mis-associated SNIa.

3.2. *Core Collapse contamination*

The potentially more pressing issue in photometric samples is core collapse contamination. Spectroscopic surveys enjoy the boon of simultaneous classification - with survey quality spectroscopy, it is easy to discern SNIa from core collapse supernovae (CC SNe). Without that, surveys are susceptible to passing along CC SNe to the Hubble diagram and subsequently to cosmological measurements. A CC contamination of even 2% can lead to a 10% shift in recovered w (Jones et al. 2018). This necessitates the use of classifiers to identify the transients we've observed throughout the survey, as well as a more thorough understanding of core collapse supernovae. As it stands, there is a dearth of knowledge about CC SNe. I have investigated the effect that this lack of understanding can have on our measurements. By changing the available core collapse templates for our classifier training set, I characterised the effect of incomplete core collapse libraries on w and found it had less than a 1% effect on measurements of w (Popovic et al. 2019). Hubble residuals, the difference between predicted cosmology and data, are shown in histogram form in figure 6. A single training library is used on simulated events with different libraries to draw events from; each panel represents a change in CC information.

During this process I also discovered that the largest potential contaminant to our sample was type Iax supernovae (up to 40% of all contamination), our largest systematic studied in the paper (see figure 6c). A peculiar subset of SNIa, observed Iax have a slightly different spectral signal (lower ejecta velocity) and are redder and dimmer than SNIa at similar distances. Despite this, currently available templates model Iax with colour and luminosity distributions similar to SNIa. This discovery has resulted in significant work being dedicated at DES to understanding and improving our Iax models.

3.3. *SNIa correlations and BBC7D*

Smith et al. (2020) further investigated the correlation of SNIa properties and their host galaxy mass. They found that simulating these correlations broke the bias corrections process developed by Kessler & Scolnic (2017) and they recovered a mass step biased by 50% from their simulated input value. It was theorised that the strong correlation of x_1 and host galaxy stellar mass was being mistaken as a mass step contribution. By

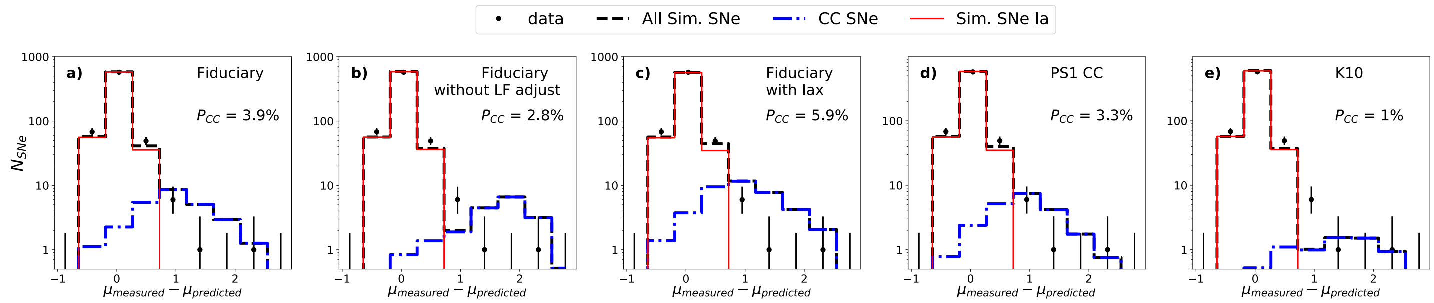

FIG. 6.— The Hubble residual distribution for data, and for simulations using different CC models as indicated on each panel. $\mu_{measured}$ is the measured Tripp distance modulus and μ_{pred} is the predicted distance modulus from ΛCDM cosmology. The training sample for the classifier is the Fiduciary CC model in all cases. Panel a) is the self consistent result, panel b) is the fiduciary set with CC luminosity adjusted, panel c) include Iax, panel d) uses the CC library in Jones et al. (2018), and panel e) uses the CC library from Kessler et al. (2010).

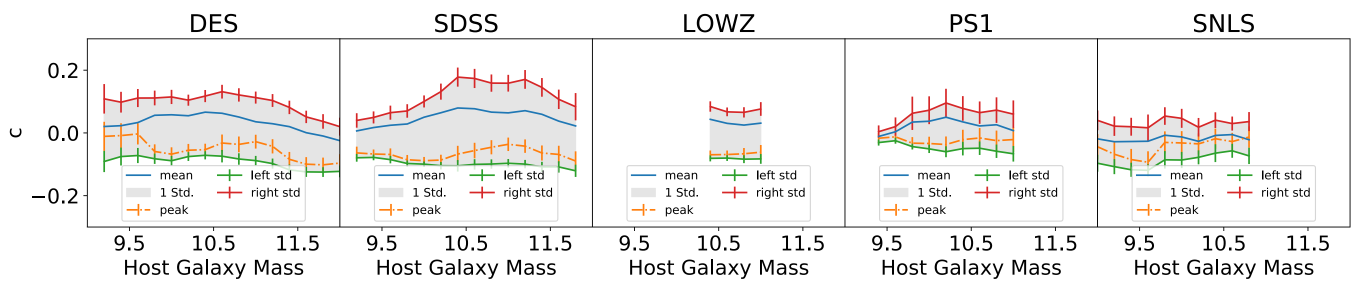

FIG. 7.— The evolution of c parent populations with host galaxy mass for several surveys. Parent populations are modeled with an asymmetric Gaussian. The mean of this Gaussian is shown in blue; grey fill shows 1 standard deviation away for both left and right hand sides. Errors on these values are shown as well. The populations are mostly consistent across surveys. The statistically significant mass range changes based on survey size.

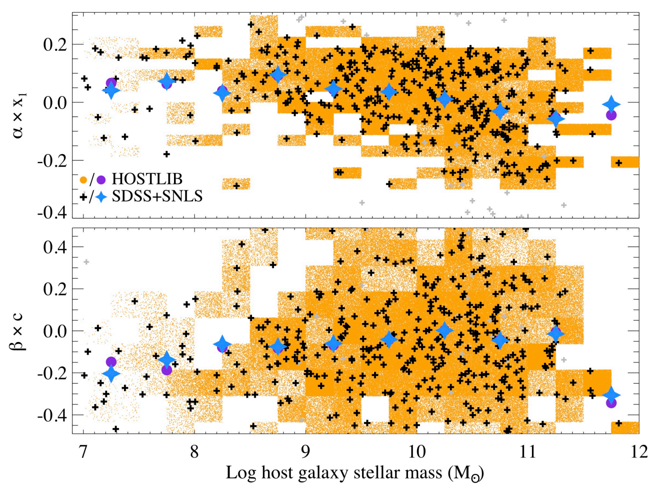

FIG. 8.— Observed $\alpha \times x_1$ and $\beta \times c$ values for the DES 3 year sample (Smith et al. 2020). Black dots are observed values from DES 3-year data, yellow boxes are simulated distributions based on the data. Blue crosses are the averaged data value and purple dots are averaged simulation values. The $\alpha \times x_1$ term shows a strong correlation with host galaxy mass.

incorporating an additional 2 dimensions in the BBC method, host galaxy mass and mass step magnitude, I designed a method to accurately recover the correct mass step while simultaneously include host galaxy correlations.

3.4. *Miscellaneous*

In the course of the DES supernova survey, 8 pairs of SNIa sharing the same host galaxy were discovered and

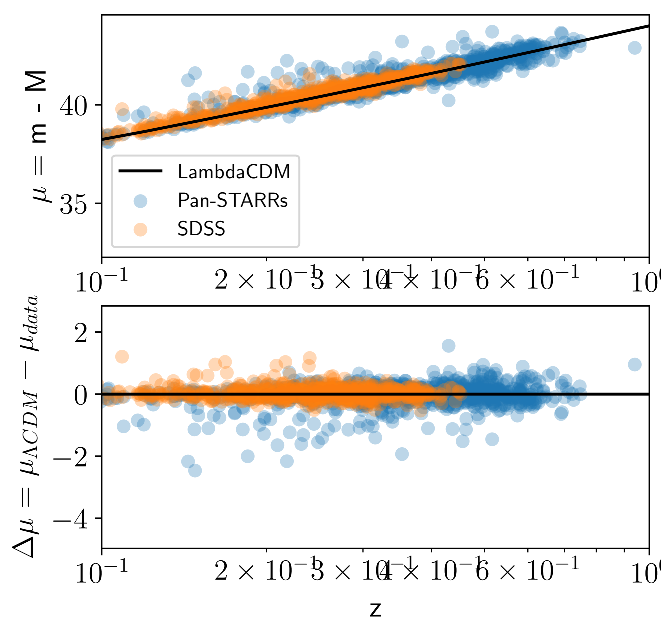

FIG. 9.— Early Hubble Diagram for the combined PS1+SDSS photometric samples. The SDSS data is classified and drawn from Popovic et al. 2020; PS1 data is unprocessed.

their properties were explored in Scolnic et al. (2020). I provided rates estimating the likelihood of detecting two supernovae in the same galaxy. I also provided aid in recent efforts at the University of Pennsylvania to investigate the robustness of our current training samples, and the impact that different spectral training sets can

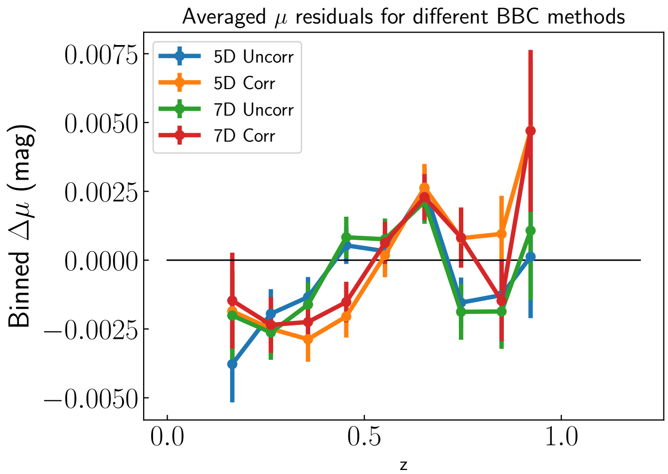

FIG. 10.— Here we compared averaged distance modulus residuals as a function of redshift for different correction methods. 'Corr' refers to including the SNIa-host galaxy correlations from Popovic et al. 2020b in simulations, 'Uncorr' does not. The residuals do not seem to be affected by using a 5D or 7D BBC process.

have on cosmological measurements.

I have also contributed ongoing efforts to the SH0ES (Supernovae, H_0, for the Equation of State of Dark Energy) collaboration, aiding in the analysis of some of the host galaxies in their supernova sample. More recently, I found a discrepancy with the average galactic reddening reported for the Tip of the Red Giant Branch (TRGB) analysis, a competing measurement of H_0.

4. FUTURE CONTRIBUTIONS

I am well prepared for work going on into the future. The DES supernova working group is currently working on the analysis for our 5 year analysis. This marks the first time that DES will be analysing a photometric sample. I am heavily involved with the project and will continue to be going forward. I have already made progress in determining the mis-assocation rate for SNIa in the DES sample and its potential impact on w. I am also involved in developing libraries of potential host galaxies for simulation analyses. We are likely to begin looking at blinded results later this year and unblinded results in 2021. The analysis will incorporate the BBC7D methodology I developed as well as several systematics from my previous works. This summer, I will be helping to lead a paper on the w impact that the BBC7D method has, in preparation for the larger DES analysis.

Concurrently, I have begun work on combining the SDSS sample from Popovic et al. (2019) with the PS1 photometric sample from Jones et al. (2018) (see figure 9). This combined sample will comprise the largest photometric supernova sample to date. The paper will include cosmological measurements on w and Ω_m and an assessment of the systematic uncertainties in the sample. The statistical uncertainty for a sample of this size is predicted to be 40% smaller than previous best efforts.

Year	Papers
Early 2020	BBC7D: Improvement of Bias Corrections of Type Ia Supernovae Distances after Accounting for Correlations with Host-galaxy Properties Impact of Spectral Features in Core Collapse Templates used to Train Photometric Supernova Classifiers BBC7D: Impact on w of Bias Corrections after Accounting for Correlations with Host-galaxy Properties
Late 2020	First Joint Photometric Analysis of confirmed Type Ia Supernovae from the combined sample of Pan-STARRs and Sloan Digital Sky Survey
Early 2021	Marginalising over mis-associated redshifts in photometric samples with z-BEAMS Help finalising DES 5 Year sample
Late 2021	DES5YR: the complete light-curve analysis of the DES 5 year photometric sample
2022	First Joint Photometric Analysis of SNIa from the combined sample of DES, Pan-STARRs, and Sloan Digital Sky Survey

In 2021, I will begin the implementation of z-BEAMS into the BBC7D method. This is a crucial step for future surveys such as LSST or WFIRST and their increased reliance on photometric samples and will serve to mitigate the potential new systematic of mis-associated redshifts. I will begin working on preparing WFIRST for supernova physics.

REFERENCES

Branch, D. & Tammann, G. A. 1992, Annual Review of Astronomy and Astrophysics, 30, 359

Gupta, R. R., Kuhlmann, S., Kovacs, E., Spinka, H., Kessler, R., Goldstein, D. A., Liotine, C., Pomian, K., D'Andrea, C. B., Sullivan, M., Carretero, J., Castander, F. J., Nichol, R. C., Finley, D. A., Fischer, J. A., Foley, R. J., Kim, A. G., Papadopoulos, A., Sako, M., Scolnic, D. M., Smith, M., Tucker, B. E., Uddin, S., Wolf, R. C., Yuan, F., Abbott, T. M. C., Abdalla, F. B., Benoit-Lévy, A., Bertin, E., Brooks, D., Carnero Rosell, A., Carrasco Kind, M., Cunha, C. E., da Costa, L. N., Desai, S., Doel, P., Eifler, T. F., Evrard, A. E., Flaugher, B., Fosalba, P., Gaztañaga, E., Gruen, D., Gruendl, R., James, D. J., Kuehn, K., Kuropatkin, N., Maia, M. A. G., Marshall, J. L., Miquel, R., Plazas, A. A., Romer, A. K., Sánchez, E., Schubnell, M., Sevilla-Noarbe, I., Sobreira, F., Suchyta, E., Swanson, M. E. C., Tarle, G., Walker, A. R., & Wester, W. 2016, AJ, 152, 154

Guy, J., Sullivan, M., Conley, A., Regnault, N., Astier, P., Balland, C., Basa, S., Carlberg, R. G., Fouchez, D., Hardin, D., Hook, I. M., Howell, D. A., Pain, R., Palanque-Delabrouille, N., Perrett, K. M., Pritchet, C. J., Rich, J., Ruhlmann-Kleider, V., Balam, D., Baumont, S., Ellis, R. S., Fabbro, S., Fakhouri, H. K., Fourmanoit, N., González-Gaitán, S., Graham, M. L., Hsiao, E., Kronborg, T., Lidman, C., Mourao, A. M., Perlmutter, S., Ripoche, P., Suzuki, N., & Walker, E. S. 2010, A&A, 523, A7

Hlozek, R., Kunz, M., Bassett, B., Smith, M., Newling, J., Varughese, M., Kessler, R., Bernstein, J. P., Campbell, H., Dilday, B., Falck, B., Frieman, J., Kuhlmann, S., Lampeitl, H., Marriner, J., Nichol, R. C., Riess, A. G., Sako, M., & Schneider, D. P. 2012, ApJ, 752, 79

Hubble, E. 1929, Proceedings of the National Academy of Sciences, 15, 168

Jones, D. O., Riess, A. G., Scolnic, D. M., Pan, Y.-C., Johnson, E., Coulter, D. A., Dettman, K. G., Foley, M. M., Foley, R. J., Huber, M. E., Jha, S. W., Kilpatrick, C. D., Kirshner, R. P., Rest, A., Schultz, A. S. B., & Siebert, M. R. 2018, ApJ, 867, 108

Kessler, R., Bernstein, J. P., Cinabro, D., Dilday, B., Frieman, J. A., Jha, S., Kuhlmann, S., Miknaitis, G., Sako, M., Taylor, M., & Vanderplas, J. 2009, Publications of the Astronomical Society of the Pacific, 121, 1028

Kessler, R., Conley, A., Jha, S., & Kuhlmann, S. 2010, arXiv e-prints, arXiv:1001.5210

Kessler, R. & Scolnic, D. 2017, ApJ, 836, 56

Leavitt, H. S. & Pickering, E. C. 1912, Harvard College Observatory Circular, 173, 1

Perlmutter, S., Aldering, G., Goldhaber, G., Knop, R. A., Nugent, P., Castro, P. G., Deustua, S., Fabbro, S., Goobar, A., Groom, D. E., Hook, I. M., Kim, A. G., Kim, M. Y., Lee, J. C., Nunes, N. J., Pain, R., Pennypacker, C. R., Quimby, R., Lidman, C., Ellis, R. S., Irwin, M., McMahon, R. G., Ruiz-Lapuente, P., Walton, N., Schaefer, B., Boyle, B. J., Filippenko, A. V., Matheson, T., Fruchter, A. S., Panagia, N., Newberg, H. J. M., Couch, W. J., & Project, T. S. C. 1999, ApJ, 517, 565

Phillips, M. M. 1993, ApJ, 413, L105

Popovic, B., Scolnic, D., & Kessler, R. 2019, arXiv e-prints, arXiv:1910.05228

Riess, A. G., Filippenko, A. V., Challis, P., Clocchiatti, A., Diercks, A., Garnavich, P. M., Gilliland, R. L., Hogan, C. J., Jha, S., Kirshner, R. P., Leibundgut, B., Phillips, M. M., Reiss, D., Schmidt, B. P., Schommer, R. A., Smith, R. C., Spyromilio, J., Stubbs, C., Suntzeff, N. B., & Tonry, J. 1998, AJ, 116, 1009

Roberts, E., Lochner, M., Fonseca, J., Bassett, B. A., Lablanche, P.-Y., & Agarwal, S. 2017, Journal of Cosmology and Astroparticle Physics, 2017, 036

Scolnic, D. & Kessler, R. 2016, ApJ, 822, L35

Scolnic, D., Kessler, R., Brout, D., Cowperthwaite, P. S., Soares-Santos, M., Annis, J., Herner, K., Chen, H.-Y., Sako, M., Doctor, Z., Butler, R. E., Palmese, A., Diehl, H. T., Frieman, J., Holz, D. E., Berger, E., Chornock, R., Villar, V. A., Nicholl, M., Biswas, R., Hounsell, R., Foley, R. J., Metzger, J., Rest, A., García-Bellido, J., Möller, A., Nugent, P., Abbott, T. M. C., Abdalla, F. B., Allam, S., Bechtol, K., Benoit-Lévy, A., Bertin, E., Brooks, D., Buckley-Geer, E., Carnero Rosell, A., Carrasco Kind, M., Carretero, J., Castander, F. J., Cunha, C. E., D'Andrea, C. B., da Costa, L. N., Davis, C., Doel, P., Drlica-Wagner, A., Eifler, T. F., Flaugher, B., Fosalba, P., Gaztanaga, E., Gerdes, D. W., Gruen, D., Gruendl, R. A., Gschwend, J., Gutierrez, G., Hartley, W. G., Honscheid, K., James, D. J., Johnson, M. W. G., Johnson, M. D., Krause, E., Kuehn, K., Kuhlmann, S., Lahav, O., Li, T. S., Lima, M., Maia, M. A. G., March, M., Marshall, J. L., Menanteau, F., Miquel, R., Neilsen, E., Plazas, A. A., Sanchez, E., Scarpine, V., Schubnell, M., Sevilla-Noarbe, I., Smith, M., Smith, R. C., Sobreira, F., Suchyta, E., Swanson, M. E. C., Tarle, G., Thomas, R. C., Tucker, D. L., Walker, A. R., & DES Collaboration. 2018, ApJ, 852, L3

Scolnic, D., Smith, M., Massiah, A., Wiseman, P., Brout, D., Kessler, R., Davis, T. M., Foley, R. J., Galbany, L., Hinton, S. R., Hounsell, R., Kelsey, L., Lidman, C., Morgan, R., Nichol, R. C., Möller, A., Popovic, B., Sako, M., Sullivan, M., Thomas, B. P., Abbott, T. M. C., Aguena, M., Allam, S., Annis, J., Avila, S., Bechtol, K., Bertin, E., Brooks, D., Burke, D. L., Carnero Rosell, A., Carollo, D., Carrasco Kind, M., Carretero, J., Costanzi, M., da Costa, L. N., De Vicente, J., Desai, S., Diehl, H. T., Doel, P., Drlica-Wagner, A., Eckert, K., Eifler, T. F., Everett, S., Flaugher, B., Fosalba, P., Frieman, J., García-Bellido, J., Gaztanaga, E., Gerdes, D. W., Glazebrook, K., Gruen, D., Gruendl, R. A., Gschwend, J., Gutierrez, G., Hartley, W. G., Hollowood, D. L., Honscheid, K., James, D. J., Kuehn, K., Kuropatkin, N., Lewis, G. F., Li, T. S., Lima, M., Macaulay, E., Maia, M. A. G., Marshall, J. L., Menanteau, F., Miquel, R., Palmese, A., Paz-Chinchón, F., Plazas, A. A., Pursiainen, M., Sanchez, E., Scarpine, V., Schubnell, M., Serrano, S., Sevilla-Noarbe, I., Sommer, N. E., Suchyta, E., Swanson, M. E. C., Tarle, G., Tucker, B. E., Varga, T. N., Walker, A. R., & Wilkinson, R. 2020, arXiv e-prints, arXiv:2002.00974

Slipher, V. M. 1917, Proceedings of the American Philosophical Society, 56, 403

Smith, M., Sullivan, M., Wiseman, P., Kessler, R., Scolnic, D., Brout, D., D'Andrea, C. B., Davis, T. M., Foley, R. J., Frohmaier, C., Galbany, L., Gupta, R. R., Gutiérrez, C. P., Hinton, S. R., Kelsey, L., Lidman, C., Macaulay, E., Möller, A., Nichol, R. C., Nugent, P., Palmese, A., Pursiainen, M., Sako, M., Thomas, R. C., Tucker, B. E., Carollo, D., Lewis, G. F., Sommer, N. E., Abbott, T. M. C., Aguena, M., Allam, S., Avila, S., Bertin, E., Bhargava, S., Brooks, D., Buckley-Geer, E., Burke, D. L., Carnero Rosell, A., Carrasco Kind, M., Costanzi, M., da Costa, L. N., De Vicente, J., Desai, S., Diehl, H. T., Doel, P., Eifler, T. F., Everett, S., Flaugher, B., Fosalba, P., Frieman, J., García-Bellido, J., Gaztanaga, E., Glazebrook, K., Gruen, D., Gruendl, R. A., Gschwend, J., Gutierrez, G., Hartley, W. G., Hollowood, D. L., Honscheid, K., James, D. J., Krause, E., Kuehn, K., Kuropatkin, N., Lima, M., MacCrann, N., Maia, M. A. G., Marshall, J. L., Martini, P., Melchior, P., Menanteau, F., Miquel, R., Paz-Chinchón, F., Plazas, A. A., Romer, A. K., Roodman, A., Rykoff, E. S., Sanchez, E., Scarpine, V., Schubnell, M., Serrano, S., Sevilla-Noarbe, I., Suchyta, E., Swanson, M. E. C., Tarle, G., Thomas, D., Tucker, D. L., Varga, T. N., Walker, A. R., :, & The Dark Energy Survey Collaboration. 2020, arXiv e-prints, arXiv:2001.11294

Sullivan, M., Conley, A., Howell, D. A., Neill, J. D., Astier, P., Balland, C., Basa, S., Carlberg, R. G., Fouchez, D., Guy, J., Hardin, D., Hook, I. M., Pain, R., Palanque-Delabrouille, N., Perrett, K. M., Pritchet, C. J., Regnault, N., Rich, J., Ruhlmann-Kleider, V., Baumont, S., Hsiao, E., Kronborg, T., Lidman, C., Perlmutter, S., & Walker, E. S. 2010, MNRAS, 406, 782

Tripp, R. 1998, A&A, 331, 815